Solutions Exam
SOLID MECHANICS (NASM)
November 2, 2022, 15:00-17:00 h

Question 1

a. Continuity of tractions on the interface translate into continuity of the stress components σ_{i2} $(i = 1, \ldots, 3)$ by virtue of $t_i = \sigma_{ij}n_j = \sigma_{i2}$.

b. No, since shear stresses can only be traced back to differences between principal stresses.

Question 2

In bending, every fiber parallel to the neutral axis is uniaxial tension, so that $U = \frac{1}{2}\int_B \sigma_{11}\varepsilon_{11}dV$, yet with both σ_{11} and ε_{11} depending linearly on x_2:

$$\sigma_{11} = E\varepsilon_{11} \text{ with } \varepsilon_{11} = \kappa x_2 = w'' x_2 \,.$$

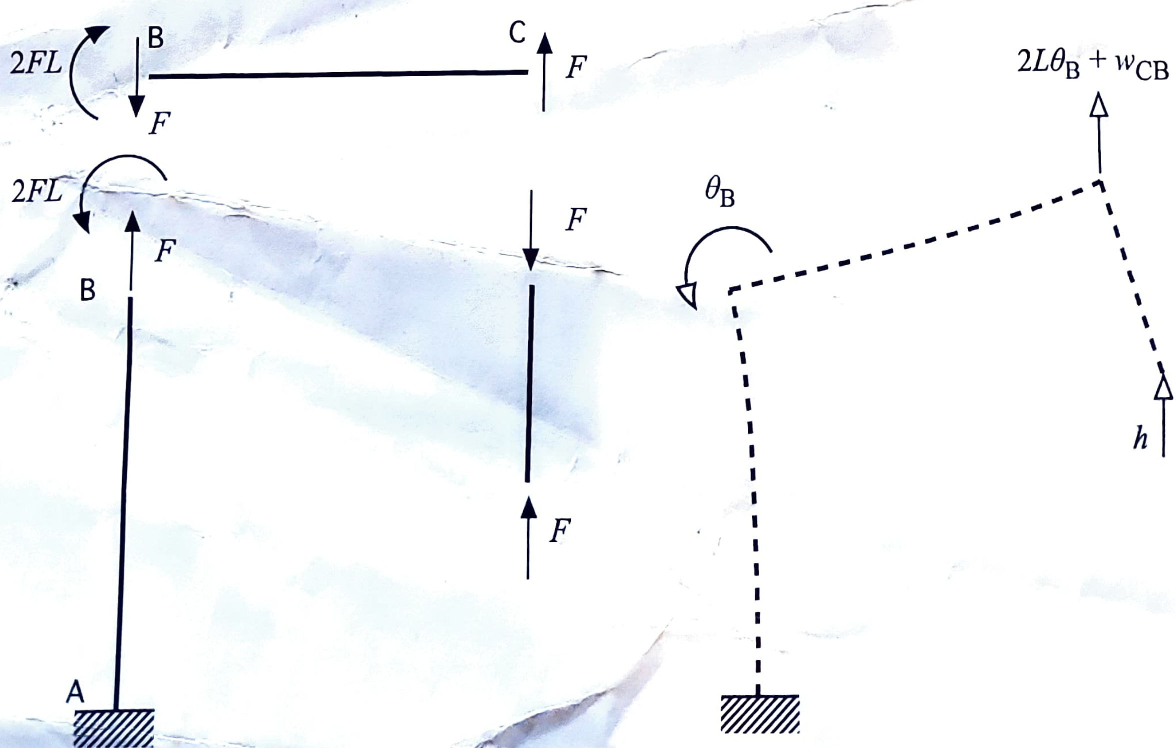
The volume integral in the foregoing expression for U can be expanded as

$$U = \frac{1}{2}\int_0^l E(w'')^2 \left\{ \int_A {x_2}^2 dx_2 dx_3 \right\} dx_1 = \frac{1}{2}\int_0^l EI(w'')^2 dx_1 \,,$$

which leads to the postulated expression in (1) when one recalls the beam bending equation $M = EIw''$ and simplifies x_1 to just x.

Question 3

a. The schematic below (left-hand side) shows the free-body diagrams of the legs of the spring. From this, we see that AB is a beam with a constant bending moment $2FL$ (forget-me-not type 1), while BC can be seen as a cantilever with end force F (forget-me-not type 2) which is rotated at B by the end rotation θ_B of AB. The remaining leg of the spring is just under a compressive force, whose elongation we can neglect. The vertical displacement $2L\theta_B + w_{BC}$ of C should therefore be equal to the opening h.



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According to the forget-me-nots,

$$\theta_B = \frac{2FL \times 2L}{EI} = \frac{4FL^2}{EI}, \quad w_{BC} = \frac{F(2L)^3}{3EI} = \frac{8FL^3}{3EI}$$

so that

$$h = 2L\theta_B + w_{BC} = \frac{8FL^3}{EI} + \frac{8FL^3}{3EI} = \frac{32FL^3}{3EI}.$$

Conversely,

$$F = \frac{3EIh}{32L^3}.$$

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b. With the aid of the beam equation (3.49) the integral can be simplified to

$$U = \int \frac{M(x)^2}{2EI} dx$$

so that we only need to know $M(x)$ and integrate:

FMN	$M(x)$	U
1	M	$\int_0^l \frac{(M)^2}{2EI} dx = \frac{M^2 l}{2EI}$
2	$F(l-x)$	$\int_0^l \frac{F^2(l-x)^2}{2EI} dx = \frac{F^2 l^3}{6EI}$

c. The results from (b) per leg of the spring:

leg	FMN	loading	U
AB	1	2FL	$\frac{4F^2L^3}{EI}$
BC	2	F	$\frac{4F^2L^3}{3EI}$

add up a total stored energy of $16F^2L^3/3EI$ which is $\equiv \frac{1}{2}Fh$ as it should for an elastic system (without friction).

Question 4

Start out by introducing the base vector, say, $\boldsymbol{e}_3$ in the vertical direction, so that the non-zero component of the stress tensor is $\sigma_{33} = \sigma$. The resolved shear stress τ_R is found by substitution of σ_{ij} into the definition of the resolved shear stress

$$\tau_R = s_i \sigma_{ij} m_j = s_3 \sigma m_3.$$

Here, m_3 is the component of the slip plane normal $\boldsymbol{m}$ in the $\boldsymbol{e}_3$ direction, i.e. $m_3 = \boldsymbol{m} \cdot \boldsymbol{e}_3 = \cos\phi$ 1
(see figure). Similarly, $s_3 = \boldsymbol{s} \cdot \boldsymbol{e}_3 = \cos\lambda$. Hence,

$$\tau_R = \sigma \cos\phi \cos\lambda.$$

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